(“ ISLAMIC”) MATHEMATICS AS ART

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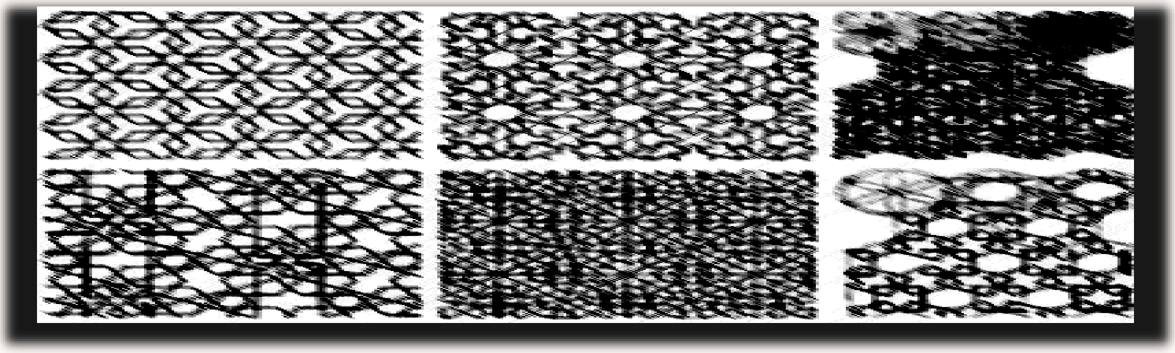
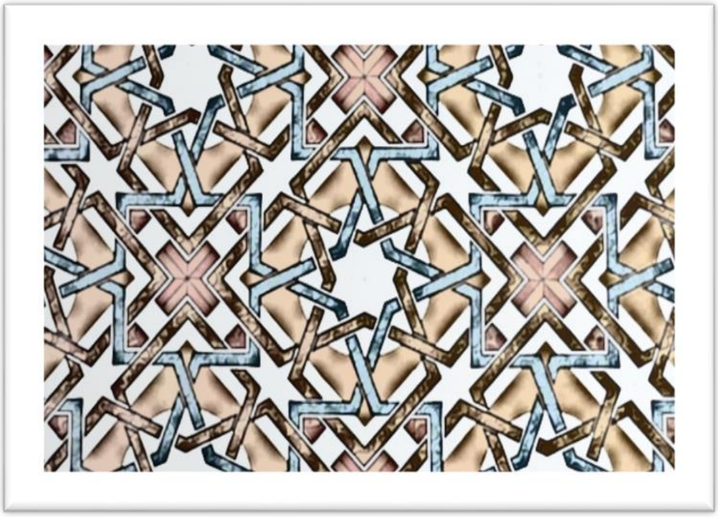
ABSTRACT

The arabesques and geometric patterns of Islamic art are often said to arise from the Islamic view of the world. These patterns are derived from Geometry, either singly or combined, adorn all types of surfaces, forming intricate and complex arrangements. While geometric ornamentation may have reached a pinnacle in the Islamic world, sources for the basic shapes and intricate patterns already existed in late antiquity. This paper examines the role mathematics played in creating what is known as ISLAMIC ART.

Foreword: Mathematics and art are related in a variety of ways. Mathematics has itself been described as an art motivated by beauty. Mathematics can be discerned in arts such as music, dance, painting, architecture, sculpture, and textiles**.** Mathematics has directly influenced art with conceptual tools such as linear perspective, the analysis of symmetry, and mathematical objects such as polyhedral and the Möbius strip**.** The Islamic Empire established across Persia, the Middle East, Central Asia, North Africa, Iberia and parts of India from the 8th Century onwards made significant contributions towards mathematics. They were able to draw and fuse together the mathematical developments of both Greece and India. One consequence of the Islamic prohibition on depicting the human form was the extensive use of complex geometric patterns to decorate their buildings, raising mathematics to the form of an art. In fact, over time, Muslim artists discovered all the different forms of symmetry that can be depicted on a 2-dimensional surface.



Islamic craftsmen turned geometry into an art form because pictures of people were not allowed in holy places. apart from the algebra, the trigonometry, the optics, the astronomy and the many other scientific advances and inventions of the Islamic Golden Age. Stunning patterns grace mosques, madrasas and palaces around the world. The mathematical elegance of these designs is that no matter how elaborate they are, they are always based on grids constructed using only a ruler and a pair of compasses. Islamic design is based on Greek geometry, which teaches us that starting with very basic assumptions, we can build up a remarkable number of proofs about shapes. Islamic patterns provide a visual confirmation of the complexity that can be achieved with such simple tools.



Introduction to Geometric Design in Islamic Art: The principles and teachings of Islam as a way of life, a religious code, and a legal system were promulgated by Muhammad (ca. 570–632 A.D.), an Arab merchant from Mecca. These teachings were revealed to him over a period of many years beginning in 610 and were subsequently codified in the text known as the Qur’an. The word of God, as set out in the Qur’an and handed down in the sayings of Muhammad (known as hadith, or Traditions), forms the core of the religion. The primary premise of the Islamic faith is monotheism, a renunciation of all deities except one, Allah, who alone is the creator, sustainer, and destroyer of life. Islam is Arabic for” submission, “here to the single entity of Allah. The recognition of Muhammad as Allah’s last prophet, a prophet like Abraham, Moses, Jesus, and the others that preceded Muhammad, is also a key element of the belief. Neither the Qur’an nor the Traditions contain specific mandates against figural representation in art. However, both sources take a firm stance against idolatry and the worship of images. These precepts were interpreted strictly by early Islamic religious leaders and exegetes as an injunction against the depiction of human or animal figures, although extant examples of architectural decoration, objects in all media, and illustrated manuscripts belie that stricture. Four types of ornamentation can be found in Islamic art: calligraphy, figural forms (human and animal), vegetal motifs, and geometric patterns.

These patterns, either singly or combined, adorn all types of surfaces, forming intricate and complex arrangements. While geometric ornamentation may have reached a pinnacle in the Islamic world, sources for the basic shapes and intricate patterns already existed in late antiquity in the Byzantine and Sasanian empires. Islamic artists appropriated key elements from the classical tradition, then elaborated upon them to invent a new form of decoration that stressed the importance of unity, logic, and order. Essential to this unique style were the contributions made by Islamic mathematicians, astronomers, and other scientists, whose ideas and technical advances are indirectly reflected in the artistic tradition. The basic instruments for constructing geometric designs were a compass and ruler. The circle became the foundation for Islamic pattern, in part a consequence of refinements made to the compass by Arabic astronomers and cartographers. The circle is often an organizing element underlying vegetal designs; it plays an important role in calligraphy, which the Arabs defined as “the geometry of the line”; and it structures all the complex Islamic patterns using geometric shapes.

The geometric designs in Islamic art are often built on combinations of repeated squares and circles, which may be overlapped and interlaced, as can arabesques (with which they are often combined), to form intricate and complex patterns, including a wide variety of tessellations. These may constitute the entire decoration, may form a framework for floral or calligraphic embellishments, or may retreat into the background around other motifs. The complexity and variety of patterns used evolved from simple stars and lozenges in the ninth century, through a variety of 6- to 13-point patterns by the 13th century, and finally to include also 14- and 16-point stars in the sixteenth century. Patterns occur in a variety of form in Islamic art including carpets known as kilims, Persian girih and Moroccan zellige tilework, muqarnas decorative vaulting, jali pierced stone screens, ceramics, leather, stained glass, woodwork, and metalwork. Interest in Islamic geometric patterns is increasing in the West, both among craftsmen and artists including M. C. Escher in the twentieth century, and among mathematicians and physicists including Peter J. Lu and Paul Steinhardt who controversially claimed in 2007 that tiling at the Darb-e Imam shrine in Isfahan could generate quasi-periodic patterns like Penrose tiling.

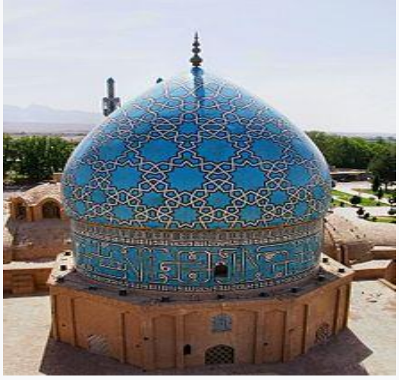
These patterns have three basic characteristics:

1. They are made up of a small number of **repeated geometric** elements. The simple forms of the circle, square, and straight line are the basis of the patterns. These elements are combined, duplicated, interlaced, and arranged in intricate combinations. Most patterns are typically based on one of two types of grid—one composed of equilateral triangles, the other of squares. A third type of grid, composed of hexagons, is a variation on the triangular schema. The mathematical term for these grids is “regular tessellation” (deriving from Latin tesserae, i.e., pieces of mosaic), in which one regular polygon is repeated to tile the plane.

2. They are **two-dimensiona**l. Islamic designs often have a background and foreground pattern. The placement of pattern upon pattern serves to flatten the space, and there is no attempt to create depth. Vegetal patterns are may be set against a contrasting background in which the plant -like forms interlace, weaving over and under in a way that emphasizes the foreground decoration. In other instances, the background is replaced by a contrast between light and shade. Sometimes it is impossible to distinguish between foreground and background. Some geometric designs are created by fitting all the polygonal shapes together like the pieces of a puzzle, leaving no gaps and, therefore, requiring no spatial interplay between foreground and background. The mathematical term for this type of construction is “tessellation.” The conception of space in Islamic art is completely different from Western models, which usually adopt a linear perspective and divide the picture space into foreground, middle ground, and background. Artists of the Islamic world were largely uninterested in linear perspective. Of the various styles of Islamic art, it was in Persian painting that a type of three-dimensional space was used in which figures could interact, but this space presented multiple viewpoints and simultaneously featured bird’s-eye and worm’s-eye views.

3. They **are not designed to fit within a frame**. Geometric ornamentation in Islamic art suggests a remarkable degree of freedom. The complex arrangements and combinations of elements are infinitely expandable; the frame surrounding a pattern appears to be arbitrary and the basic arrangement sometimes provides a unit from which the rest of the design can be both predicted and projected.

Pattern formation



The Shah Nematollah Vali Shrine, Mahan, Iran, 1431. The blue girih-tiled dome contains stars with, from the top, 5, 7, 9, 12, 11, 9 and 10 points in turn. 11-point stars are rare in Islamic art.

Further information: Mathematics and art and Girih

Many Islamic designs are built on squares and circles, typically repeated, overlapped and interlaced to form intricate and complex patterns. A recurring motif is the 8-pointed star, often seen in Islamic tilework; it is made of two squares, one rotated 45 degrees with respect to the other. The fourth basic shape is the polygon, including pentagons and octagons. All of these can be combined and reworked to form complicated patterns with a variety of symmetries including reflections and rotations. Such patterns can be seen as mathematical tessellations, which can extend indefinitely and thus suggest infinity. They are constructed on grids that require only ruler and compass to draw. Artist and educator Roman Verostko argues that such constructions are in effect algorithms, making Islamic geometric patterns forerunners of modern algorithmic art.

The circle symbolizes unity and diversity in nature, and many Islamic patterns are drawn starting with a circle. For example, the decoration of the 15th-century mosque in Yazd, Persia is based on a circle, divided into six by six circles drawn around it, all touching at its centre and each touching its two neighbours' centres to form a regular hexagon. On this basis is constructed a six-pointed star surrounded by six smaller irregular hexagons to form a tessellating star pattern. This forms the basic design which is outlined in white on the wall of the mosque. That design, however, is overlaid with an intersecting tracery in blue around tiles of other colours, forming an elaborate pattern that partially conceals the original and underlying design. A similar design forms the logo of the Mohammed Ali Research Center.

One of the early Western students of Islamic patterns, Ernest Hanbury Hankin, defined a "geometrical arabesque" as a pattern formed "with the help of construction lines consisting of polygons in contact." He observed that many different combinations of polygons can be used as long as the residual spaces between the polygons are reasonably symmetrical. For example, a grid of octagons in contact has squares (of the same side as the octagons) as the residual spaces. Every octagon is the basis for an 8-point star, as seen at Akbar's tomb, Sikandra (1605–1613). Hankin considered the "skill of the Arabian artists in discovering suitable combinations of polygons almost astounding." He further records that if a star occurs in a corner, exactly one quarter of it should be shown; if along an edge, exactly one half of it.

TOPKAPI PALACE

On the lacquered ceiling of the throne, studded with jewels, are foliage patterns accompanied by the depiction of the fight of a dragon, symbol of power, with *simurg*, a mythical bird. On the throne there is a cover made of several pieces of brocade on which emerald and ruby plaques and pearls are sown. Embossed inscriptions at the main visitors' door, dating from 1856, contain laudatory words for Sultan Abdülmecid I. The main door is surmounted by an embossed *besmele*, the common Muslim benediction, meaning "In the Name of God the Compassionate, the Merciful", dating from 1723. This inscription was added during the reign of Sultan Ahmed III. The tile panels on either side of the door were placed during later repair work

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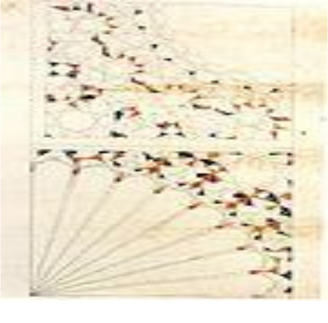
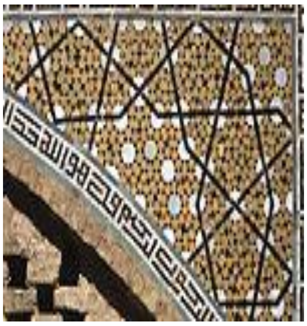
The Tiled Pavilion is the earliest building of Topkapi Palace, built by Mehmet II (the Conqueror). The striking tiles which adorn the entire building still display strong traces of Seljuk Turkish art in both the designs and the predominance of blue and turquoise. It is for this reason that the building has been transformed into a ceramics museum, where the finest examples of Turkish ceramics from the 12th century to the present day are on display. At the entrance to G'lhane Park is the Alay K÷sk' (meaning Ceremonial Pavilion) dating from the reign of Mahmud II (1808-1839) who watched various parades and processions from this vantage point.

The Topkapı Scroll, made in Timurid dynasty Iran in the late-15th century or beginning of the 16th century, contains 114 patterns including coloured designs for girih tiling and muqarnas quarter or semi domes.

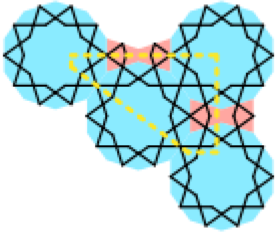
The mathematical properties of the decorative tile and stucco patterns of the Alhambra palace in Granada, Spain have been extensively studied. Some authors have claimed on dubious grounds to have found most or all of the 17 wallpaper groups there. Moroccan geometric woodwork from the 14th to 19th centuries makes use of only 5 wallpaper groups, mainly p4mm and c2mm, with p6mm and p2mm occasionally and p4gm rarely; it is claimed that the "Hasba" (measure) method of construction, which starts with *n*-fold rosettes, can however generate all 17 groups.

- 

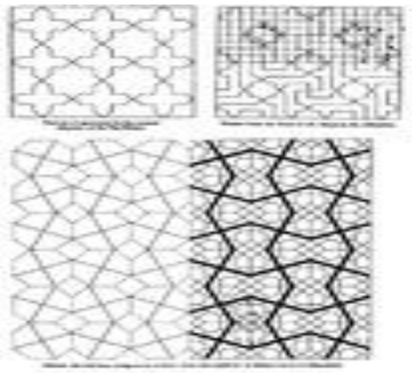
Two-dimensional designs for two quarter-dome muqarnas – as a seashell (top), as a fan (bottom). Topkapı Scroll, 15th century



Girih tiling in the decagonal pattern on a spandrel from the Darb-e Imam shrine

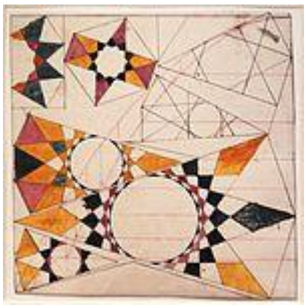


Construction of girih pattern in Darb-e Imam spandrel (yellow line). Construction decagons blue, bowties red. The strapwork cuts across the construction tessellation.



Analysis of octagonal patterns in Mughal architecture by Ernest Hanbury Hankin, 1925. 8-pointed stars emerge (lower right) where heavy black lines cross.

Decoration in Tomb of I'timād-ud-Daulah, Agra, showing correct treatment of sides and corners. A quarter of each 6-point star is shown in each corner; half stars along the sides.



Architectural drawing for brick vaulting, Iran, probably Tehran, 1800–70 [1]

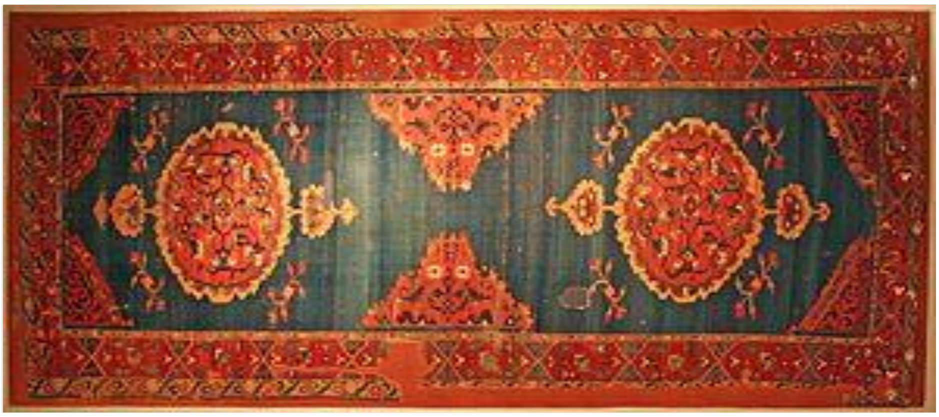
The four basic shapes, or “repeat units,” from which the more complicated patterns are constructed are: circles and interlaced circles; squares or four-sided polygons; the ubiquitous star pattern, ultimately derived from squares and triangles inscribed in a circle; and multisided polygons. It is clear, however, that the complex patterns found on many objects include a number of different shapes and arrangements, allowing them to fit into more than one category.

regional influence and the prevailing lifestyles during ruling dynasties determined the diversity of Islamic ornaments and geometrical patterns. For example, basic 6- and 8-point geometrical patterns, introduced during the late 9th century, are the most pervasive Islamic ornaments. Aside from their originality, the simplicity in construction of these patterns drove architects to use such ornaments in almost all building elements, from floor finishes to minaret surfaces. Whilst the difficulty of the abstract and the complexity of non-constructible geometrical patterns limited their application to accessible elements (Qibla walls, window screens), particularly in Iran and central Asia. Another interesting result is that in contrast to the architects and artisans from other Islamic states, those from Anatolia paid less attention to ornaments and geometrical patterns; they focused more extensively on other aspects of architecture, such as form and master planning. For this reason, only a few examples of complex and sophisticated patterns (aside from the simplest ones) can be found in Anatolia.

The relatively stable government and economy during the Mamluk period encouraged architects to design very fine and detailed ornaments that are unique in terms of complexity. The intricate 16-point patterns remained popular in North Africa and Islamic Spain, but only minimally influenced eastern regions, such as Persia, Anatolia, and the Mughal region.

Simpler patterns were popular in the Indian subcontinent, which may be attributed to the passion of Indian artisans for symmetrical designs and their insistence on covering all exterior surfaces with ornaments. Such coverage would be difficult to achieve when complex patterns are used. For centuries, Islamic geometrical patterns (IGPs) have been used as decorative elements on walls, ceilings, doors, domes, and minarets. However, the absence of guidelines and codes on the application of these ornaments often leads to inappropriate use in terms of time scale accuracy and architectural matching.

Planar symmetries



Powerful presence: carpet with double medallion. Central Anatolia (Konya – Karapınar), turn of the 16th/17th centuries. Alâeddin Mosque

Planar symmetries have for millennia been exploited in artworks such as carpets, lattices, textiles and tiling. Many traditional rugs, whether pile carpets or flatweave kilims, are divided into a central field and a framing border; both can have symmetries, though in handwoven carpets these are often slightly broken by small details, variations of pattern and shifts in colour introduced by the weaver. In kilims from Anatolia, the motifs used are themselves usually symmetrical. The general layout, too, is usually present, with arrangements such as stripes, stripes alternating with rows of motifs, and packed arrays of roughly hexagonal motifs. The field is commonly laid out as a wallpaper with a wallpaper group such as pmm, while the border may be laid out as a frieze of frieze group pm11, pmm2 or pma2.

Turkish and Central Asian kilims often have three or more borders in different frieze groups. Weavers certainly had the intention of symmetry, without explicit knowledge of its mathematics. The mathematician and architectural theorist Nikos Salingaros suggests that the "powerful presence" (aesthetic effect) of a "great carpet" such as the best Konya two-medallion carpets of the 17th century is created by mathematical techniques related to the theories of the architect Christopher Alexander. These techniques include making opposites couple; opposing colour values; differentiating areas geometrically, whether by using complementary shapes or balancing the directionality of sharp angles; providing small-scale complexity (from the knot level upwards) and both small- and large-scale symmetry; repeating elements at a hierarchy of different scales (with a ratio of about 2.7 from each level to the next). Salingaros argues that "all successful carpets satisfy at least nine of the above ten rules", and suggests that it might be possible to create a metric from these rules. Elaborate lattices are found in Indian Jali work, carved in marble to adorn tombs and palaces they often have mirror, double mirror, or rotational symmetry. Some have a central medallion, and some have a border in a frieze group.

Girih tiles

Islamic art exploits symmetries in many of its artforms, notably in girih tiling. These are formed using a set of five tile shapes, namely a regular decagon, an elongated hexagon, a bow tie, a rhombus, and a regular pentagon. All the sides of these tiles have the same length; and all their angles are multiples of 36° (π/5 radians), offering fivefold and tenfold symmetries. The tiles are decorated with strapwork lines (girih), generally more visible than the tile boundaries. In 2007, the physicists Peter Lu and Paul Steinhardt argued that girih resembled quasi-crystalline Penrose tiling. Elaborate geometric zellige tilework is a distinctive element in Moroccan architecture. Muqarnas vaults are three-dimensional but were designed in two dimensions with drawings of geometrical cells

Symmetries are prominent in textile arts including quilting, knitting, cross-stitch, crochet, embroidery and weaving, where they may be purely decorative or may be marks of status. Rotational symmetry of Islamic designs is found in circular structures such as domes; these are sometimes elaborately decorated with symmetric patterns inside and out, as at the 1619 Sheikh Lotfollah Mosque in Isfahan. Items of leafy embroidery and lace work such as tablecloths and table mats, made using bobbins or by tatting, can have a wide variety of reflectional and rotational symmetries which are being explored mathematically.

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The visual intricacy of Islamic mathematical structures have inspired a variety of artwork -mathematical and otherwise.

REFERENCES

1. Freely adopted from Wikipedia.